

# Direct $CP$ violation and isospin triangles of $B \rightarrow \pi\pi$ decays

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The recent observation of  $B_d^0 \rightarrow \pi^0 \pi^0$  and  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  decay modes allows us to make a fresh isospin analysis of  $B \rightarrow \pi\pi$  transitions. We find that current experimental data can impose some model-independent constraints on the parameter space of direct  $CP$  violation in  $B_d \rightarrow \pi^+ \pi^-$  and  $B_d \rightarrow \pi^0 \pi^0$ . Furthermore, we establish a direct relationship between the weak phase  $\alpha$  and the charge-averaged branching fractions and  $CP$ -violating asymmetries of  $B \rightarrow \pi\pi$  decays.

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The major goal of  $B$ -meson factories is to test the Kobayashi-Maskawa mechanism of  $CP$  violation within the standard model and to detect possible new sources of  $CP$  violation beyond the standard model. So far the  $CP$ -violating asymmetry in  $B_d^0$  vs  $\bar{B}_d^0 \rightarrow J/\psi K_S$  decays has unambiguously been measured at KEK and SLAC [1], and the experimental result is very compatible with the standard-model expectation. Some preliminary evidence for  $CP$  violation in a number of other  $B$  decays has also been reported by BaBar and Belle Collaborations [2]. Further experiments will provide much more precise data on  $CP$  violation in the  $B$ -meson system, from which one may cross-check the consistency of the Kobayashi-Maskawa picture and probe possible new physics.

The charmless two-body nonleptonic decays  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ ,  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  and  $B_u^- \rightarrow \pi^0 \pi^-$ , which can be related to one another via the isospin triangle [3], have been of great interest in  $B$  physics for a stringent test of the factorization hypothesis, a quantitative analysis of final-state interactions, and a clean determination of the  $CP$ -violating phase  $\alpha \equiv \arg[-(V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)]$  [an inner angle of the well-known unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [4]]. Among these three decay modes,  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  is color suppressed and should have the smallest branching fraction. The observation of this rare decay has recently been reported by BaBar [5] and Belle [6] Collaborations:<sup>1</sup>

$$\mathcal{B}_{00} \equiv \frac{\mathcal{B}(B_d^0 \rightarrow \pi^0 \pi^0) + \mathcal{B}(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)}{2} = \begin{cases} (2.1 \pm 0.6 \pm 0.3) \times 10^{-6} & (\text{BaBar}) \\ (1.7 \pm 0.6 \pm 0.3) \times 10^{-6} & (\text{Belle}), \end{cases} \quad (1)$$

where the first error is statistical and the second one is systematic. The experimental value of  $\mathcal{B}_{00}$  is a bit larger than the

theoretical prediction in various QCD models [7]. In comparison, the observed branching fractions of  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$  and  $B_u^- \rightarrow \pi^0 \pi^-$  decays [8]

$$\mathcal{B}_{+-} \equiv \frac{\mathcal{B}(B_d^0 \rightarrow \pi^+ \pi^-) + \mathcal{B}(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)}{2} = \begin{cases} (4.7 \pm 0.6 \pm 0.2) \times 10^{-6} & (\text{BaBar}) \\ (4.4 \pm 0.6 \pm 0.3) \times 10^{-6} & (\text{Belle}) \\ (4.5 \pm 1.4 \pm 0.5) \times 10^{-6} & (\text{CLEO}) \end{cases} \quad (2)$$

and

$$\mathcal{B}_{0\pm} \equiv \frac{\mathcal{B}(B_u^+ \rightarrow \pi^0 \pi^+) + \mathcal{B}(B_u^- \rightarrow \pi^0 \pi^-)}{2} = \begin{cases} (5.5 \pm 1.0 \pm 0.6) \times 10^{-6} & (\text{BaBar}) \\ (5.3 \pm 1.3 \pm 0.5) \times 10^{-6} & (\text{Belle}) \\ (4.6 \pm 1.8 \pm 0.7) \times 10^{-6} & (\text{CLEO}) \end{cases} \quad (3)$$

are essentially consistent with the theoretical results [7]. Because  $\mathcal{B}_{+-}$ ,  $\mathcal{B}_{00}$  and  $\mathcal{B}_{0\pm}$  are all charge averaged, they cannot directly be related to one another through the isospin relation.

The purpose of this paper is to make a fresh isospin analysis of three  $B \rightarrow \pi\pi$  decays by using new experimental data and taking account of  $CP$  violation. We find that current data can impose some useful and model-independent constraints on the parameter space of direct  $CP$  violation in  $B_d \rightarrow \pi^+ \pi^-$  and  $B_d \rightarrow \pi^0 \pi^0$  transitions. It is also possible to establish a direct relationship between the weak phase  $\alpha$  and the charge-averaged branching fractions and  $CP$ -violating asymmetries of  $B \rightarrow \pi\pi$  decays.

Let us define the *direct*  $CP$ -violating asymmetries between  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ ,  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$ ,  $B_u^- \rightarrow \pi^0 \pi^-$  and their  $CP$ -conjugate decays:

$$\mathcal{A}_{+-} \equiv \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow \pi^+ \pi^-) - \mathcal{B}(B_d^0 \rightarrow \pi^+ \pi^-)}{\mathcal{B}(\bar{B}_d^0 \rightarrow \pi^+ \pi^-) + \mathcal{B}(B_d^0 \rightarrow \pi^+ \pi^-)},$$

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<sup>1</sup>Note that the experimentally reported branching fractions of  $B \rightarrow \pi\pi$  decays are all charge averaged.

$$\mathcal{A}_{00} \equiv \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow \pi^0 \pi^0) - \mathcal{B}(B_d^0 \rightarrow \pi^0 \pi^0)}{\mathcal{B}(\bar{B}_d^0 \rightarrow \pi^0 \pi^0) + \mathcal{B}(B_d^0 \rightarrow \pi^0 \pi^0)},$$

$$\mathcal{A}_{0\pm} \equiv \frac{\mathcal{B}(B_u^- \rightarrow \pi^0 \pi^-) - \mathcal{B}(B_u^+ \rightarrow \pi^0 \pi^+)}{\mathcal{B}(B_u^- \rightarrow \pi^0 \pi^-) + \mathcal{B}(B_u^+ \rightarrow \pi^0 \pi^+)}. \quad (4)$$

Taking account of Eqs. (1)–(4), one can easily obtain the magnitude of each decay amplitude,

$$|A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)| \propto \sqrt{\frac{\mathcal{B}_{+-}}{\tau_0}} (1 + \mathcal{A}_{+-}),$$

$$|A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)| \propto \sqrt{\frac{\mathcal{B}_{00}}{\tau_0}} (1 + \mathcal{A}_{00}),$$

$$|A(B_u^- \rightarrow \pi^0 \pi^-)| \propto \sqrt{\frac{\mathcal{B}_{0\pm}}{\tau_{\pm}}} (1 + \mathcal{A}_{0\pm}); \quad (5)$$

and

$$|A(B_d^0 \rightarrow \pi^+ \pi^-)| \propto \sqrt{\frac{\mathcal{B}_{+-}}{\tau_0}} (1 - \mathcal{A}_{+-}),$$

$$|A(B_d^0 \rightarrow \pi^0 \pi^0)| \propto \sqrt{\frac{\mathcal{B}_{00}}{\tau_0}} (1 - \mathcal{A}_{00}),$$

$$|A(B_u^+ \rightarrow \pi^0 \pi^+)| \propto \sqrt{\frac{\mathcal{B}_{0\pm}}{\tau_{\pm}}} (1 - \mathcal{A}_{0\pm}), \quad (6)$$

where  $\tau_0$  and  $\tau_{\pm}$  denote the lifetimes of neutral and charged  $B$  mesons, respectively. The present experimental data yield  $\kappa \equiv \tau_0/\tau_{\pm} \approx 0.92$  [4]. In Eqs. (5) and (6), we have neglected the tiny phase space difference between  $\pi^0 \pi^{\pm}$  and  $\pi^+ \pi^-$  (or  $\pi^0 \pi^0$ ) states. It is obvious that the relevant decay amplitudes cannot be determined, unless the  $CP$ -violating asymmetries  $\mathcal{A}_{+-}$ ,  $\mathcal{A}_{00}$  and  $\mathcal{A}_{0\pm}$  are all measured.

Under isospin symmetry and in the neglect of electroweak penguin contributions [9], the amplitudes of  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ ,  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  and  $B_u^- \rightarrow \pi^0 \pi^-$  decays form an isospin triangle in the complex plane:

$$A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-) + \sqrt{2}A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0) = \sqrt{2}A(B_u^- \rightarrow \pi^0 \pi^-). \quad (7)$$

Similarly,

$$A(B_d^0 \rightarrow \pi^+ \pi^-) + \sqrt{2}A(B_d^0 \rightarrow \pi^0 \pi^0) = \sqrt{2}A(B_u^+ \rightarrow \pi^0 \pi^+). \quad (8)$$

In this approximation, the magnitudes of  $A(B_u^- \rightarrow \pi^0 \pi^-)$  and  $A(B_u^+ \rightarrow \pi^0 \pi^+)$  are identical to each other [3]. Thus the  $CP$ -violating asymmetry  $\mathcal{A}_{0\pm}$  vanishes. What we are concerned about is the relative phase between the amplitudes of  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$  (or  $B_d^0 \rightarrow \pi^+ \pi^-$ ) and  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  (or  $B_d^0 \rightarrow \pi^0 \pi^0$ ) decays:

$$\phi \equiv \arg \left[ \frac{A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)}{A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)} \right],$$

$$\varphi \equiv \arg \left[ \frac{A(B_d^0 \rightarrow \pi^+ \pi^-)}{A(B_d^0 \rightarrow \pi^0 \pi^0)} \right]. \quad (9)$$

At the tree level,  $\varphi = \phi$  holds. A difference between  $\phi$  and  $\varphi$  measures the QCD penguin effects in  $B_d \rightarrow \pi^+ \pi^-$  and  $B_d \rightarrow \pi^0 \pi^0$  transitions. With the help of Eqs. (5)–(8), we find

$$\cos \phi = \frac{2\kappa\mathcal{B}_{0\pm} - \mathcal{B}_{+-}(1 + \mathcal{A}_{+-}) - 2\mathcal{B}_{00}(1 + \mathcal{A}_{00})}{2\sqrt{2\mathcal{B}_{+-}\mathcal{B}_{00}}(1 + \mathcal{A}_{+-})(1 + \mathcal{A}_{00})},$$

$$\cos \varphi = \frac{2\kappa\mathcal{B}_{0\pm} - \mathcal{B}_{+-}(1 - \mathcal{A}_{+-}) - 2\mathcal{B}_{00}(1 - \mathcal{A}_{00})}{2\sqrt{2\mathcal{B}_{+-}\mathcal{B}_{00}}(1 - \mathcal{A}_{+-})(1 - \mathcal{A}_{00})}, \quad (10)$$

where  $\mathcal{A}_{0\pm} = 0$  has been taken into account. One can see that  $\varphi = \phi$  would hold, if both  $\mathcal{A}_{+-}$  and  $\mathcal{A}_{00}$  were vanishing. Hence the difference between  $\phi$  and  $\varphi$  results from the penguin-induced  $CP$  violation in  $B_d \rightarrow \pi^+ \pi^-$  and  $B_d \rightarrow \pi^0 \pi^0$  decays.

To get a ball-park feeling of the discrepancy between  $\phi$  and  $\varphi$ , we make use of the QCD factorization to estimate the ratio of  $A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)$  to  $A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)$  and that of  $A(B_d^0 \rightarrow \pi^+ \pi^-)$  to  $A(B_d^0 \rightarrow \pi^0 \pi^0)$  [10]. The results are

$$\frac{A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)}{A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)} = \frac{(a_1 + a_4^u + a_6^u \xi_{\pi})R_b e^{-i\gamma} + (a_4^c + a_6^c \xi_{\pi})}{(a_2 - a_4^u - a_6^u \xi_{\pi})R_b e^{-i\gamma} - (a_4^c + a_6^c \xi_{\pi})},$$

$$\frac{A(B_d^0 \rightarrow \pi^+ \pi^-)}{A(B_d^0 \rightarrow \pi^0 \pi^0)} = \frac{(a_1 + a_4^u + a_6^u \xi_{\pi})R_b e^{+i\gamma} + (a_4^c + a_6^c \xi_{\pi})}{(a_2 - a_4^u - a_6^u \xi_{\pi})R_b e^{+i\gamma} - (a_4^c + a_6^c \xi_{\pi})}, \quad (11)$$

where  $R_b \equiv |V_{ud}V_{ub}^*|/|V_{cd}V_{cb}^*|$  and  $\gamma \equiv \arg[-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)]$  are two parameters of the CKM unitarity triangle [11];  $a_1$ ,  $a_2$ ,  $a_4^{u,c}$  and  $a_6^{u,c}$  denote the effective Wilson coefficients [12];  $\xi_{\pi}$  is associated with the penguin operators  $Q_{5,6}$  [13] and can be given as

$$\xi_{\pi} = \frac{2m_{\pi}^2}{(m_b - m_u)(m_u + m_d)} \quad (12)$$

under isospin symmetry. For illustration, we typically adopt  $R_b \approx 0.37$  and  $\gamma \approx 65^\circ$  [11] as well as  $a_1 \approx 1.038 + 0.018i$ ,  $a_2 \approx 0.082 - 0.080i$ ,  $a_4^u \approx -0.029 - 0.015i$  and  $a_4^c \approx -0.034 - 0.008i$  at the scale  $\mu = m_b$  [12]. The formally power-suppressed QCD coefficients  $a_6^{u,c}$  can be neglected in the heavy quark limit [12], because of  $\xi_{\pi} \sim \mathcal{O}(1)$ . Then we arrive at  $A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)/A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0) \approx 7.4e^{-i17^\circ}$  and  $A(B_d^0 \rightarrow \pi^+ \pi^-)/A(B_d^0 \rightarrow \pi^0 \pi^0) \approx 4.3e^{+i44^\circ}$ . As a result,  $\phi \approx -17^\circ$  and  $\varphi \approx 44^\circ$  are obtained. The big difference  $\varphi - \phi \approx 61^\circ$  implies the existence of significant QCD penguin effects and large direct  $CP$  violation in  $B_d \rightarrow \pi\pi$  decays. It should be noted that  $\phi$  and  $\varphi$  are neither pure weak phases

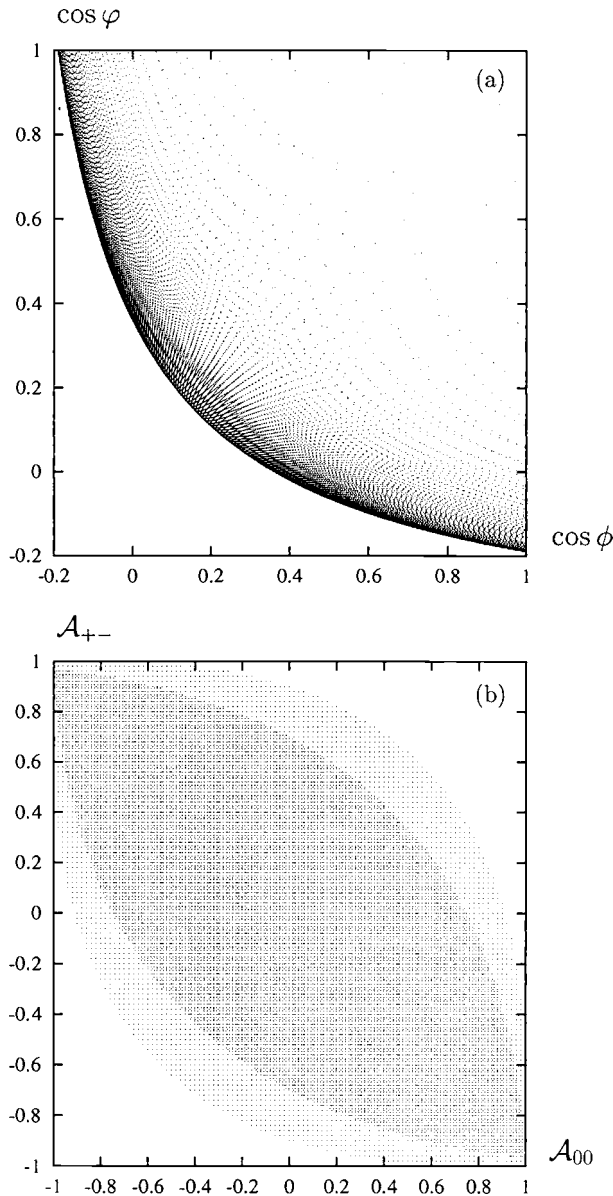


FIG. 1. Constraints of current experimental data on (a) the parameter space of  $\cos \phi$  and  $\cos \varphi$  and (b) the parameter space of  $\mathcal{A}_{+-}$  and  $\mathcal{A}_{00}$ . Note that the dark region of (b) corresponds to the central values of  $B_{+-}$ ,  $B_{00}$  and  $B_{0\pm}$  given in Eqs. (1)–(3).

nor pure strong phases, since the tree and penguin amplitudes of each  $B_d \rightarrow \pi\pi$  decay mode involve different weak and strong phases. It should also be noted that the numerical results for  $\phi$  and  $\varphi$  are just illustrative, because possible final-state rescattering effects have not been taken into account in Eq. (11).

Once the  $CP$ -violating asymmetries  $\mathcal{A}_{+-}$  and  $\mathcal{A}_{00}$  are measured at  $B$  factories, it will be possible to determine  $\phi$  and  $\varphi$  from Eq. (10) independent of the specific QCD models. A hint of  $\mathcal{A}_{+-} \neq 0$  (of  $\sim 2\sigma$  significance) has recently been reported by Belle Collaboration, but BaBar Collaboration's preliminary result for  $\mathcal{A}_{+-}$  is consistent with zero [2]. There is no experimental information about  $\mathcal{A}_{00}$  at present. In this situation, we show the numerical correlation between  $\phi$  and  $\varphi$  in Fig. 1(a) and that between  $\mathcal{A}_{+-}$  and  $\mathcal{A}_{00}$  in Fig.

1(b), where current data listed in Eqs. (1)–(3) have been used. The points in Fig. 1(a) are generated from scanning the error bars of  $B_{+-}$ ,  $B_{00}$  and  $B_{0\pm}$ . The light region in Fig. 1(b) is generated in a similar way and corresponds to the allowed ranges of  $\cos \phi$  and  $\cos \varphi$  in Fig. 1(a), while the dark region in Fig. 1(b) results from inputting the central values of  $B_{+-}$ ,  $B_{00}$  and  $B_{0\pm}$ . Some comments and discussions are in order.

(1) Current experimental results of  $B_{+-}$ ,  $B_{00}$  and  $B_{0\pm}$  can impose some limited but model-independent constraints on the values of  $(\phi, \varphi)$  and  $(\mathcal{A}_{+-}, \mathcal{A}_{00})$ . For example, the possibilities  $\cos \phi = \cos \varphi = 0$  and  $|\mathcal{A}_{+-}| = |\mathcal{A}_{00}| = 1$  are not allowed. As a consequence of  $|\mathcal{A}_{+-}| < 1$  and  $|\mathcal{A}_{00}| < 1$ , the regions for  $\cos \phi < -0.2$  and  $\cos \varphi < -0.2$  have completely been excluded by the present experimental data.

(2) The error bars of  $B_{+-}$ ,  $B_{00}$  and  $B_{0\pm}$  influence the parameter space of  $(\phi, \varphi)$  or  $(\mathcal{A}_{+-}, \mathcal{A}_{00})$  to a very limited extent. If  $\mathcal{A}_{+-}$  is measured, one can get a roughly allowed range of  $\mathcal{A}_{00}$  from Fig. 1(b). For instance,  $\mathcal{A}_{+-} \geq 0.6$  and  $\mathcal{A}_{00} \geq 0.6$  cannot simultaneously hold nor can  $\mathcal{A}_{+-} \leq -0.6$  and  $\mathcal{A}_{00} \leq -0.6$  hold.

(3) As the tree-level amplitude of  $B_d^0 \rightarrow \pi^0 \pi^0$  or  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  is color suppressed, its magnitude might be comparable with the size of the corresponding penguin amplitude. Then  $|\mathcal{A}_{00}|$  is expected to be larger than  $|\mathcal{A}_{+-}|$  in the QCD factorization. However, the elastic rescattering effects in the final states of  $B \rightarrow \pi\pi$  transitions could easily spoil this naive expectation. It is therefore useful to analyze the relevant experimental data in a model-independent way.

(4) The validity of our numerical results relies on the validity of the isospin relations in Eqs. (7) and (8), which hold in the assumption of negligible electroweak penguin effects. The electroweak penguin contributions to  $B \rightarrow \pi\pi$  transitions are generally expected to be insignificant [9]. This expectation would be problematic or incorrect, if  $\mathcal{A}_{0\pm} \neq 0$  were experimentally established. It is also worth mentioning that final-state interactions in  $B \rightarrow \pi\pi$  decays include both elastic  $\pi\pi \rightleftharpoons \pi\pi$  rescattering and some possible inelastic rescattering effects.<sup>2</sup> Whether inelastic final-state interactions are negligibly small or not remains an open question [16]. To answer this question requires more precise data on both the branching fractions and the  $CP$ -violating asymmetries of  $B \rightarrow \pi\pi$  decays.

We proceed to discuss the extraction of the  $CP$ -violating phase  $\alpha$  with the help of the isospin triangles in Eqs. (7) and (8). As shown in Ref. [3],  $\alpha$  depends on two  $CP$ -violating observables which arise from the interplay of decay and  $B_d^0$ - $\bar{B}_d^0$  mixing in  $B_d \rightarrow \pi^+ \pi^-$  and  $B_d \rightarrow \pi^0 \pi^0$  decays:

<sup>2</sup>The amplitudes of  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ ,  $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$  and  $B_u^- \rightarrow \pi^0 \pi^-$  decay modes (or their  $CP$ -conjugate processes) may still form an isospin triangle in the complex plane, even if the inelastic  $\pi\pi \rightleftharpoons D\bar{D}$  rescattering effects are taken into account [14]. In this specific case, the analytical result obtained in Eq. (10) remains valid. It is even expected that inelastic final-state interactions could account for the existing discrepancy between experimental data and QCD models for  $D \rightarrow \pi\pi$  and  $B \rightarrow \pi\pi$  decays [15].

$$\chi_{+-} \equiv \text{Im} \left[ \frac{q}{p} \cdot \frac{A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)}{A(B_d^0 \rightarrow \pi^+ \pi^-)} \right] = |R| \sin[2(\alpha + \Theta)],$$

$$\chi_{00} \equiv \text{Im} \left[ \frac{q}{p} \cdot \frac{A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)}{A(B_d^0 \rightarrow \pi^0 \pi^0)} \right] = |\bar{R}| \sin[2(\alpha + \bar{\Theta})], \quad (13)$$

where

$$R = |R| e^{2i\Theta} = \frac{1 - \bar{r}}{1 - r},$$

$$\bar{R} = |\bar{R}| e^{2i\bar{\Theta}} = \frac{2 + \bar{r}}{2 + r}, \quad (14)$$

with  $r = |r| e^{i\theta} \equiv A_0/A_2$  and  $\bar{r} = |\bar{r}| e^{i\bar{\theta}} \equiv \bar{A}_0/\bar{A}_2$  being the ratios of  $I=0$  and  $I=2$  isospin amplitudes in  $B_d^0 \rightarrow \pi^+ \pi^-$  (or  $\pi^0 \pi^0$ ) and  $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$  (or  $\pi^0 \pi^0$ ) decays. Following Ref. [17], we obtain

$$|r| = \sqrt{3(a+b)-2},$$

$$\theta = \pm \arccos \left[ \frac{6b-3a-2}{4\sqrt{3(a+b)-2}} \right]; \quad (15)$$

and

$$|\bar{r}| = \sqrt{3(\bar{a}+\bar{b})-2},$$

$$\bar{\theta} = \pm \arccos \left[ \frac{6\bar{b}-3\bar{a}-2}{4\sqrt{3(\bar{a}+\bar{b})-2}} \right], \quad (16)$$

where

$$a \equiv \left| \frac{A(B_d^0 \rightarrow \pi^+ \pi^-)}{A(B_u^+ \rightarrow \pi^0 \pi^+)} \right|^2 = \frac{1}{\kappa} \cdot \frac{\mathcal{B}_{+-}}{\mathcal{B}_{0\pm}} (1 - \mathcal{A}_{+-}),$$

$$b \equiv \left| \frac{A(B_d^0 \rightarrow \pi^0 \pi^0)}{A(B_u^+ \rightarrow \pi^0 \pi^+)} \right|^2 = \frac{1}{\kappa} \cdot \frac{\mathcal{B}_{00}}{\mathcal{B}_{0\pm}} (1 - \mathcal{A}_{00}); \quad (17)$$

and

$$\bar{a} \equiv \left| \frac{A(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)}{A(B_u^- \rightarrow \pi^0 \pi^-)} \right|^2 = \frac{1}{\kappa} \cdot \frac{\mathcal{B}_{+-}}{\mathcal{B}_{0\pm}} (1 + \mathcal{A}_{+-}),$$

$$\bar{b} \equiv \left| \frac{A(\bar{B}_d^0 \rightarrow \pi^0 \pi^0)}{A(B_u^- \rightarrow \pi^0 \pi^-)} \right|^2 = \frac{1}{\kappa} \cdot \frac{\mathcal{B}_{00}}{\mathcal{B}_{0\pm}} (1 + \mathcal{A}_{00}). \quad (18)$$

Clearly  $r$  (or  $\bar{r}$ ) can be determined up to a twofold ambiguity in the sign of its phase, if both  $\mathcal{A}_{+-}$  and  $\mathcal{A}_{00}$  are measured. One can in turn determine  $R$  (or  $\bar{R}$ ), both its magnitude and its phase. It is then possible to extract the weak phase  $\alpha$  from Eq. (13) through the time-dependent measurement of  $\chi_{+-}$  and (or)  $\chi_{00}$  in  $B_d \rightarrow \pi^+ \pi^-$  and (or)  $B_d \rightarrow \pi^0 \pi^0$  decays. On the  $Y(4S)$  resonance, the time-dependent rate asymmetry between  $B_d^0(t)$  and  $\bar{B}_d^0(t)$  decays into a  $CP$  eigenstate  $f$  reads as

$$\Delta_f(t) \equiv \frac{\Gamma[\bar{B}_d^0(t) \rightarrow f] - \Gamma[B_d^0(t) \rightarrow f]}{\Gamma[\bar{B}_d^0(t) \rightarrow f] + \Gamma[B_d^0(t) \rightarrow f]}$$

$$= \frac{|\rho_f|^2 - 1}{|\rho_f|^2 + 1} \cos(\Delta M_d \cdot t) + \frac{2\text{Im}\left(\frac{q}{p} \rho_f\right)}{|\rho_f|^2 + 1} \sin(\Delta M_d \cdot t), \quad (19)$$

where  $\rho_f \equiv A(\bar{B}_d^0 \rightarrow f)/A(B_d^0 \rightarrow f)$  and tiny  $CP$  violation in  $B_d^0 - \bar{B}_d^0$  mixing has been neglected (i.e.,  $|q/p| \approx 1$ ) [18]. For  $f = \pi^+ \pi^-$  and  $\pi^0 \pi^0$ , we take account of Eqs. (4)–(6) and (13) and obtain

$$\Delta_{\pi^+ \pi^-}(t) = \mathcal{A}_{+-} \cos(\Delta M_d \cdot t)$$

$$+ (1 - \mathcal{A}_{+-}) \chi_{+-} \sin(\Delta M_d \cdot t),$$

$$\Delta_{\pi^0 \pi^0}(t) = \mathcal{A}_{00} \cos(\Delta M_d \cdot t)$$

$$+ (1 - \mathcal{A}_{00}) \chi_{00} \sin(\Delta M_d \cdot t). \quad (20)$$

Clearly both  $\mathcal{A}_{+-}$  (or  $\mathcal{A}_{00}$ ) and  $\chi_{+-}$  (or  $\chi_{00}$ ) can be determined from measuring the time distribution of  $\Delta_{\pi^+ \pi^-}$  (or  $\Delta_{\pi^0 \pi^0}$ ) at asymmetric  $B$  factories. Compared with Refs. [3,17], the formulas obtained in Eqs. (13)–(18) and (20) establish a more *direct* relationship between  $\alpha$  and the observables  $\mathcal{B}_{+-}$ ,  $\mathcal{B}_{00}$ ,  $\mathcal{B}_{0\pm}$ ,  $\mathcal{A}_{+-}$  and  $\mathcal{A}_{00}$ . Thus the usefulness of our results is noteworthy.

To summarize, we have presented a fresh isospin analysis of rare  $B \rightarrow \pi\pi$  decays by taking account of the fact that the experimentally reported branching fractions are charge averaged and large  $CP$  violation may exist in them. We find that current experimental data can impose some useful and model-independent constraints on the parameter space of direct  $CP$  violation in  $B_d \rightarrow \pi^+ \pi^-$  and  $B_d \rightarrow \pi^0 \pi^0$  transitions. A straightforward relationship between the  $CP$ -violating phase  $\alpha$  and the measurables of  $B \rightarrow \pi\pi$  decays has also been established. More precise measurements of such charmless  $B$  decays will allow us to test the consistency of the Kobayashi-Maskawa mechanism of  $CP$  violation and probe possible new physics beyond the standard model.

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